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Land-sparing vs Land-sharing with incomplete policies

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Abstract

This article analyzes the trade-off between yield and farmed area when a valuable species is affected by agricultural practices. It revisits, from an economic perspective, the "land-sparing versus land-sharing" debate elaborated in conservation biology using the methodology of the density-yield curve: The density of the species on farmland is a decreasing function of the yield. It is shown that the optimal yield is either increasing or decreasing with respect to the value of the species depending on the shape of the density-yield curve. Land-sparing and land-sharing are not necessarily antagonistic; for sufficiently elastic demand function, both the optimal yield and the farmed area decrease with the value of the species. A general assessment of a second-best policy is performed, and several particular policies are considered, including a subsidy on biodiversity in farms, a tax or subsidy on farmland, and a tax or subsidy on a dirty input. In several cases, the first-best strategy and the second-best one induce contrasting effects on the yield.

JEL Classification: H23; Q15; Q57

Keywords: Biodiversity; Agriculture; Second-best policy.

1 Introduction

Agriculture is arguably the human activity with the largest impact on biodiversity (Green et al.; 2005; Pereira et al.; 2012; Tilman et al.; 2017). The environmental footprint of agriculture comes both from farming practices and from the conversion of land to agriculture. Approximately one third of the Earth's ice-free land surface is used for agricultural production (Ramankutty et al.; 2008).[1] There is a trade-off at the heart of lively debates between farming intensity (and associated environmental degradation) and the total land farmed. Since a seminal article by Green et al. (2005) this debate has been framed among conservation scientists as a choice between two extreme strategies: land-sparing and land-sharing. In a land-sparing strategy farming is concentrated on the smallest possible area the rest being spared for nature; In a land-sharing strategy, "wildlife friendly" agriculture, with low yield and better in-farm environmental quality, is performed over a large surface.

Green et al. (2005) develop a framework to analyze the trade-off between yield and farmed area from a conservationist point of view. They introduce the density-yield curve: the relationship between species density (number of specimen per hectare) and agricultural yield (production per hectare), and maximize a species abundance subject to a food production constraint. If the density yield curve is everywhere convex or concave, they find that the optimal strategy for species conservation is one of the extremes, either land-sparing or land-sharing. Land-sparing is optimal for a concave density yield curve, and land-sharing for a convex one. Green et al. (2005)'s article triggered a controversy, fueled by numerous articles, in the ecology literature (see Fischer et al.; 2014, for an attempt to "move forward").

The objective of the present article is to analyze the trade-off between yield and farmed area in an economic framework encompassing consumer surplus and production costs. The model used is a partial equilibrium model of the market for an agricultural good produced in the habitat of a valuable species. The total size of the habitat is split between farmed and unfarmed land. The density of the species per hectare is a function of the yield. The optimal welfare-maximizing yield and production are described. The optimal yield is between the

[1] Agricultural land is the sum of arable land (12%) and pastures (22%). A pedagogical presentation of figures can be found at: `https://ourworldindata.org/yields-and-land-use-in-agriculture`.

laissez-faire yield and the conservation-optimal yield; it is either higher or lower than the laissez-faire yield, and it is either increasing or decreasing with respect to the value of the species, approaching the conservation optimum.

The introduction of the demand for food allows us to consider its adjustment. Indeed, food consumption decreases when the value of the species is internalized. The demand price elasticity determines whether the total farmed area actually increases when the yield decreases. If demand for food is elastic, both lower yield and larger unfarmed area are optimal when the density-yield curve is concave. A result that shows that land-sparing and land-sharing are not mutually exclusive strategies.

The article then considers the policy consequences in a second-best setting. Indeed, the first-best optimal allocation can be implemented with a policy that rewards land-owners for the value of biodiversity both on farmed and unfarmed lands (e.g. Klimek et al.; 2008). However, such policy might not be available. For instance, an agri-environmental scheme that only reward biodiversity on farms still constitute a subsidy to farming even though not a subsidy to production (e.g. Kleijn et al.; 2006, for a description of few European schemes). The analysis of the second-best policy highlights that whether a particular agricultural practice should be promoted depends on the policy instrument available. For instance, even if the agricultural yield should be increased in the first-best solution it should not necessarily be subsidized in a second best setting in which natural reserves cannot be enforced.

Whether a policy is welfare enhancing will depend on the shape of the density-yield curve and the elasticity of the demand function.[2] For instance, even if land-sharing (reduced yield and increased area) is optimal in a first-best setting, it may be welfare enhancing to implement natural reserves if the demand is elastic. In such a case, the quantity of food consumed should decrease sufficiently to ensure that the biodiversity gained on unfarmed land compensates for the loss from higher yield on farmed land.

The density yield curve is a key methodological innovation of Green et al. (2005), but few estimates exist. Phalan, Onial, Balmford and Green (2011) construct density-yield curves

[2]The role of the elasticity of demand has been mentioned in several articles (e.g. Green et al.; 2005; Phalan, Balmford, Green and Scharlemann; 2011; Lambin and Meyfroidt; 2011), but not considered analytically in an integrated framework.

for bird and tree species in southwest Ghana and northern India.[3] They conclude that land-sparing is the optimal strategy (see also Balmford et al.; 2015). However, land-sparing has been criticized notably for the potential difficulty of implementing it. Once land has been irreversibly converted to intensive farming, it may be difficult to enforce the actual sparing of the remaining land (Godfray; 2011; Ewers et al.; 2009). The present analysis of the second-best policy partly addresses this concern. If land-sparing cannot be effectively enforced, subsidizing intensive farming induces an over-expansion of farming (compared to the first-best policy), which can compensate for the benefits from an increased yield. This negative result arises if the demand for food is sufficiently price elastic. If the demand for food is inelastic then subsidizing intensive farming enhances welfare.[4]

Green et al. (2005) are not the first to argue that intensive agriculture can be good for the environment despite its local environmental cost by sparing land for nature (Waggoner; 1996; Borlaug; 2002, e.g.). The land-sparing vs land-sharing debate on the potential benefits of intensive farming echoes the debate on the environmental consequences of the green revolution and the associated intensification of agriculture in some developing countries (see Paarlberg; 2013, Chapter 6, for a brief exposition). Some authors have studied whether an increase of the yield (as an exogenous shock) actually spares land for nature (e.g., Rudel et al.; 2009; Ewers et al.; 2009). The causes of deforestation have also been analyzed both theoretically and empirically.[5] Whether an increase of agricultural productivity leads to a decrease of farmed area depends notably upon the price elasticity of demand. This empirical question is different from the optimal way to internalize biodiversity value. The optimal policy should be concerned with both agricultural practices and farmed area and not focus

[3] The relationship between agricultural practices and some species densities has been investigated (Fuller et al.; 2005; Chamberlain et al.; 2010; Firbank et al.; 2008). In particular, the comparison of organic and conventional agriculture has received considerable attention. Many studies conclude that organic farming enhances biodiversity on farms, even though some species might be adversely affected (see the meta-analysis by Bengtsson et al.; 2005). Most studies also conclude that the yield of organic farming is lower than the yield of conventional farming, but the results are highly variable (de Ponti et al.; 2012; Seufert et al.; 2012).

[4] See Muhammad et al. (2011) for estimates of the demand for food in different countries. The demand for food is lower in wealthier countries, mainly because of the revenue effect (Slutsky elasticity is U-shaped).

[5] See (Angelsen and Kaimowitz; 1999) for a review, and Leblois et al. (2016) for a recent empirical analysis that stresses the role of international trade.

exclusively on the former.

In addition to the previously mentioned literature on deforestation, several articles in the economic literature consider the relationship between land use and biodiversity.

Hart et al. (2014) reframe the Green et al. (2005)'s analysis as a cost minimization problem. They minimize the cost to farmers to reach a target of wild nature.[6] They show that if the cost function is everywhere convex or concave the optimal solution is land sharing or land sparing, a result similar to Green et al. (2005). For more general cost functions, they establish that intermediary efforts could be optimal for a subset of farmers.[7] They apply their framework to the analysis of bird protection in mown grasslands in Sweden. In the present article, the food demand is explicitly modeled, the optimal yield is also intermediary because of the fixed cost associated to farming and not to nature protection, and the cost to further increase yield beyond the laissez-faire situation is considered.

Desquilbet et al. (2017) stress the role of agricultural markets but consider only two farming systems (intensive and extensive farming). They compare the environmental and market outcomes with the two systems, they stress the negative environmental consequences associated to the rebound effect. The equilibrium production is larger with intensive farming than with extensive farming because of its lower production cost and the elasticity of demand. Consequently, even with a convex density-yield curve biodiversity can be higher with extensive farming rather than intensive farming. They further compare consumers surplus and producer profits. Our result that even if the density yield curve is convex intensive farming should not be subsidized if demand is sufficiently elastic, shares similarities with their results. However, the converse also hold, even with a concave density yield curve extensive farming should not be subsidized if the demand is elastic.

Martinet (2013) considers two types of farming and introduces heterogeneous land productivity. He analyzes the food and wildlife production possibility set. Because of heterogeneous soil quality, and the possibility to reallocate production from less productive to more

[6]Hart et al. (2014) analyze the dual problem of the problem considered by Green et al. (2005). The density yield curve of Green et al. (2005) corresponds to the cost function of Hart et al. (2014) which could be interpreted as a profit loss on an agricultural market in which the price is implicitly assumed fixed.

[7]The case of more general density yield curves is also briefly considered in the supplementary material of Green et al. (2005), and the graphical discussion therein parallel the analysis of Hart et al. (2014).

productive land, he shows that biodiversity associated to a given food production might be maximized with a coexistence of intensive farming on the most productive land, extensive farming on intermediate productivity land, and natural reserve on the less productive land. This situation can arise when the implicit density yield curve is concave (a case corresponding to land-sharing in the framework of Green et al.; 2005). In the present article, farmers are assumed identical (as in Green et al.; 2005), and the introduction of heterogeneity as Martinet (2013) is path for future research.

Less related literature include the work of Eichner and Pethig (2006) in which a general equilibrium of the economy is linked to a general equilibrium of an ecosystem (Tschirhart; 2000). In their model, land is either used for human activity or for wildlife; there is no intermediate level (see also Christiaans et al.; 2007; Pethig; 2004, on pesticide uses). They do not analyze the trade-off between the area used and the intensity of human activity. However, a natural extension of the present work would be to develop the biological side of the model in the spirit of these works.

The rest of the article is organized as follows. The model is introduced (Section 2. The optimal (first-best) policy is described in Section 3, and second-best policies are described in section 4. The main limitations of the model are discussed in Section 5. Section 6 concludes.

2 Model

The model is voluntarily kept as simple as possible in order to encompass the framework of Green et al. (2005), namely the density-yield curve, into an economic model with a variable total production. We consider the market for one food product. The total quantity produced is F (in t.). This creates the gross consumer surplus $S(F)$ (in \$), a positive increasing and concave function. The corresponding price function $P(F)$, equal to S', is positive and decreasing. The price elasticity of the demand for food is denoted ϵ:

$$\epsilon(F) = \frac{P}{P'(F)F}. \tag{1}$$

On the supply side, the yield is denoted y (in t/ha) and the quantity of land farmed L (in ha) so that $F = yL$. The cost of farming is $c(y)$ (\$/ha); this is the cost to produce y

tons of food on a hectare of land. The total cost to produce F is then $c(y)L = c(y)F/y$. The cost $c(y)$ is positive, increasing and convex.[8] It is assumed that there is a fixed cost associated with land conversion, $c(0) > 0$, so that average costs are first decreasing and then increasing. This fixed cost can also be interpreted as the opportunity cost of farming. The cost-minimizing yield, given a fixed total production F, is denoted y_0 and is the solution of

$$c'(y) = c(y)/y. \tag{2}$$

The marginal and the average cost are equalized at y_0, which is the minimum efficient scale.

The market is assumed perfectly competitive. If no regulation is implemented, the total profit from land-use is

$$\pi = pyL - c(y)L = pF - \frac{c(y)}{y}F. \tag{3}$$

If land is abundant (the constraint on land is not binding), farmers choose the yield y_0 per hectare farmed, and the quantity of land farmed L_0 is such that the price of food is equal to the cost $c(y_0)/y_0$. The yield y_0 will be called the *laissez-faire yield* to stress that it is the yield chosen without any regulation.[9]

There is one valuable species, and the size of its population on a particular piece of land is a function of the yield.[10] The total size of the habitat and potential area of farmed land is $\bar{L}$. The density on a hectare of farmland is $b(y)$ (specimen/ha), a positive and decreasing function of the yield.[11] The total population on the habitat under consideration is the sum of the population on unfarmed land $b(0)(\bar{L} - L)$ and farmed land $b(y)L$. The marginal value of biodiversity is β ($/specimen).

[8] Convexity is related to decreasing return to scale on an hectare, and not to land heterogeneity as Desquilbet et al. (2013) and Martinet (2013).

[9] It is possible to consider an equivalent decentralized process in which each owner of a piece of land decides whether to farm or not (entry stage) and then chooses its production y (production stage). With this decentralized process, y is such that $p = c'(y)$ (price equals marginal cost), and the entry process ensures that profits are null: $py = c(y)$ so that at equilibrium $y = y_0$ and $p = c(y_0)/y_0$.

[10] We do not consider the issues of habitat fragmentation and the spatial distribution of farming activities (Lewis and Plantinga; 2007; Lewis et al.; 2009). This is one of the recurrent criticism addressed to the density-yield curve methodology (Fischer et al.; 2014).

[11] Phalan, Onial, Balmford and Green (2011) empirically establish these relationships for several bird species. They find that some species might benefit from an increased yield, a possibility not considered here.

Total welfare is then

$$W = S(yL) - c(y)L + \beta[b(0)(\bar{L} - L) + b(y)L],$$

which can be written as a function of yield and food production:

$$W(y, F) = S(F) - \left[\frac{c(y)}{y} + \beta\frac{b(0) - b(y)}{y}\right] F + \beta b(0)\bar{L}. \tag{4}$$

To ensure that there is a unique interior optimum, the value of the species is assumed sufficiently small so that $c''(y) > \beta b''(y)$ for all y. This condition is always satisfied if the density-yield curve is concave and the species is valuable. If the density-yield curve is convex and β is large, it does not hold. In that case, welfare is not concave with respect to the yield, and there can be several local optima. We rule out this possibility.

Only density yield curve convex or concave everywhere are considered in order to focus on the influence of demand and costs functions. More complex curves (e.g., first convex then concave) are possible and would be more realistic. There is a maximum yield $\bar{y}$ at which b is null, and production cost is sufficiently large at this yield that it is never optimal to adopt it.

The model is kept as simple as possible to identify the mechanisms at stake. It could be interpreted in more or less abstract ways. In particular, the function b could be seen as an aggregate biodiversity indicator and the yield as an agricultural technique ranging from intensive farming to organic farming, through various agri-environmental possibilities. A more direct interpretation of y as a direct function of fertilizer choice is elaborated in subsection 4.4. The introduction of other inputs and proper time consideration is left for further research even though briefly discussed in section 5.

3 The optimal solution

The optimal solution consists of a pair of yield and quantity of food $(y^*(\beta), F^*(\beta))$ that maximizes welfare subject to the constraint $L \leq \bar{L}$. If the constraint is not binding, the optimal policy is characterized by the two first-order conditions:[12]

[12] The assumption on β: $c'' < \beta b''$ ensures that welfare is quasi-concave. Let us show that if y and F satisfy the pair of conditions 5 and 6, then the second-order conditions are satisfied. This is because i) $\partial W/\partial y$ is

$$P(F) = \frac{c(y)}{y} + \beta \frac{b(0) - b(y)}{y} \tag{5}$$

$$c(y) - c'(y)y = \beta \left[-b'(y)y - (b(0) - b(y)) \right]. \tag{6}$$

The first equation states that the price of food should be equalized with its marginal cost, which includes the environmental cost of agriculture. The second equation represents the arbitrage made when choosing the optimal yield between economic benefits and environmental damages. The left-hand side of equation (6) is the gain due to the influence of the yield on the cost. Interestingly, with linear valuation of biodiversity the optimal yield does not depend on the demand for food.[13]

It is decreasing and null at the minimum efficient scale y_0. The right-hand side is the environmental cost from an increase in the yield. This environmental cost is the difference between the direct costs associated with the increase of the yield and the indirect gain obtained from the reduction of the farmland area ($b(0) - b(y)$ per ha). Because both sides of the equation (6) can be positive or negative, the arbitrage is not easy to appreciate.

Proposition 1 *The quantity of food produced decreases with respect to the value of the species* β*.*

If land is abundant, i.e., $F^* < y^*\bar{L}$*, the optimal yield decreases (resp. increases) with respect to* β *if* $b(y)$ *is concave (resp. convex).*

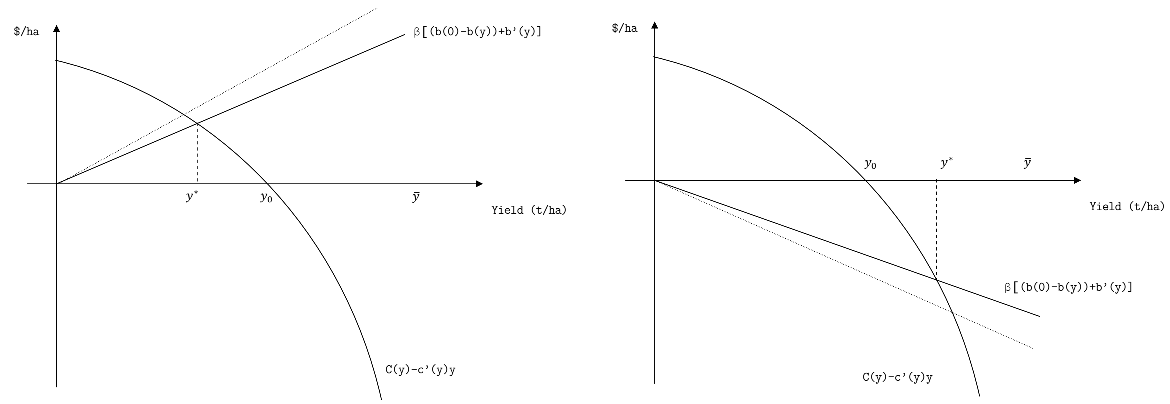
If land is scarce, i.e., $F^* = y^*\bar{L}$*, the optimal yield is decreasing with respect to* β*.*

The proof is in Appendix. The proposition is illustrated by Figure 1. With a concave density-yield curve (Figure 1(a)), the situation looks familiar: there is increasing environmental damage associated with an increase of the yield, and the optimal yield should be lower than the laissez-faire yield. The case of the convex density-yield curve is different, as

linear in F, so $\partial W^2/\partial y \partial F = [\partial W/\partial y]/F$ is null if y satisfies the equation (6), and ii) the second-order derivative w.r.t. to the yield is $\partial W^2/\partial y^2 = F/y^2(c'' - \beta b'') - 2F/y \partial W/\partial y = F/y^2(c'' - \beta b'') < 0$ if y satisfies 6).

[13] It is why it is handier to work with (y, F) rather than (y, L). If the value of biodiversity is $B\big(b(o)[\bar{L} - L) + b(y)L]\big)$ with $B(.)$ concave, the marginal value of biodiversity depends on the quantity produced and an increase in the demand for food induces a higher biodiversity marginal value and corresponding yield.

illustrated in Figure 1(b). With a convex density-yield curve, the species is very sensitive to the first increase of the yield. The marginal environmental damage is decreasing with respect to the yield, and the environmental cost is actually a gain due to the sparing of land. It is optimal to spare land in which the species is abundant and increase yield on farmland.[14]



(a) With a concave density-yield curve, the optimal yield is lower than the efficient minimum scale

(b) With a convex density-yield curve, the optimal yield is higher than the efficient minimum scale

Figure 1: The determination of the optimal yield with a concave (resp. convex) density-yield curve. The dotted line represents the effect of an increase of the value of the species.

In Figure 1, it is assumed that land is abundant for farming so that any change of the yield for a given food production is associated with an adjustment of the farmed area. If land is scarce and already fully exploited, a marginal increase in the yield does not trigger a reduction in the area farmed. In such a case, a marginal increase in the value of the species β induces a reduction in the yield regardless of the shape of the density-yield curve.

Proposition 1 illustrates that the optimal yield depends on what is held fixed. If one considers a fixed amount of land, it is always optimal to reduce the yield and adopt a wildlife-friendly technique becasue it is then the only way to increase biodiversity. However,

[14] If the density-yield curve is convex, the environmental damage is concave, and the right-hand side of 6 is decreasing with respect to the yield. The environmental cost is actually a gain that is increasing with respect to the yield, and multiple local optima may exist for a sufficiently large value of β. All interior optima are on the right side of the minimum efficient scale. When the value of the species β increases, one may jump from one interior equilibrium to another. Proposition 1 is still true because the new equilibrium is situated to the right of the old one.

if the adjustment of the farmed area is considered, the optimal yield is either increasing or decreasing with respect to the value of the species depending on the shape of the density-yield curve. For the rest of the article, the amount of land will be considered sufficiently large so that in all cases considered the constraint on $\bar{L}$ is not binding and both natural reserves and farming coexist.

In their article, Green et al. (2005) compute the optimal strategy that maximize abundance subject to a constraint on the quantity of food produced and the quantity of land available. Absent any cost consideration, it can be seen in Figure 1 that the optimal strategy is in a corner; it is optimal to set the lowest (resp. highest) possible yield with a concave (resp. convex) density-yield curve. Furthermore, their implicit demand for food is totally inelastic ($\epsilon = 0$). With an elastic demand for food, the quantity of food produced is reduced by the internalization of biodiversity, and the total farmed area depends upon demand elasticity.

Proposition 2 *The optimal farmed area is decreasing with respect to the value of the species. unless the density-yield curve is concave and the price elasticity of the demand function is higher than*

$$\frac{c(y^*) + \beta(b(0) - b(y^*))}{\beta(b(0) - b(y^*))} \times \frac{\beta(b'y^* + b(0) - b(y^*))}{(c'' - \beta b'')y^{*2}}(< 0) \tag{7}$$

Even if the optimal yield decreases with respect to the value of biodiversity, the optimal quantity of farmland can decrease if demand is sufficiently price elastic. The threshold price elasticity (7) is the product of two factors. The first is positive; it is the inverse of the β elasticity of the total production cost or, equivalently, the relative weight of the environmental component in the farming cost. The second factor is the β elasticity of the optimal yield (negative). An increase in the value of the species decreases the yield and increases the cost. The total farmed area decreases despite the reduction of the yield if the increase in the cost is sufficiently large (a small first factor in 7) to trigger a large reduction of the quantity consumed.

The result only concerns a small change in β. Larger changes might generate non-monotonic patterns. The threshold elasticity depends upon β both directly and indirectly via the yield. It is possible that the sensitivity of the yield associated with β, the second factor in (7), progressively increases toward zero, whereas the share in the cost of the environmental component increases toward unity as β increases. The two evolutions imply that the threshold elasticity is increasing toward zero, and the farmed area has a bell-shaped evolution w.r.t. to the value of the species. For initial increments, there is a strong reduction in the yield and a relatively small increase in the production cost so that the demand for food is not strongly affected and farmed area increases. For a larger value of the species, the yield is reduced less, and most environmental protection occurs via a reduction of the consumption of food and the associated farmed area.

4 Second-best policy

Having described the optimal solution, let us consider the policy implications.

The optimal allocation has been described by a yield, which could be interpreted as a farming technique, and the farmed area. This allocation can be implemented by directly setting these two quantities via technical standards and natural reserves. It can also been implemented by a Pigouvian subsidy on each species specimen equal to β and received by the farmer even in the absence of farming. The Pigouvian solution has informational advantage if the farmer is better able to determine practices that would enhance biodiversity at low cost. A feature outside the scope of the present model.

Current Agri-environmental schemes as described and anlyzed by Kleijn et al. (2006) rarely target a specific species, and only concerned farmed land. In addition to mixed environmental results (Kleijn et al.; 2006) they still constitute a subsidy to farming. Still, well designed and coupled with natural reserves there can be part of an optimal policy.

In this section, several situations are considered in which the regulator cannot implement the optimal policy. Several reasons could be proposed to explain why it is not feasible to implement the subsidy on the species specimen or to directly set the optimal yield and optimal farmed area. In addition to the difficulty of estimating the density of a species, if property rights are not well defined on unfarmed land, it is not possible to remunerate an owner to create an incentive for land conservation. A situation more likely to occur un developing countries.

4.1 A general assessment

The regulation is represented by a variable r. The regulation influences the incentive to farm and the choice of the yield; it does not have other effects. For instance, public funds are costless, and if the regulation is a tax or a subsidy, the associated monetary transfers are welfare neutral. With a regulation r , the perceived marginal cost of food production is denoted $\gamma(y,r)$. The profit of the representative land-owner is

$$\pi(y,F,P) = PF - \gamma(y,r)F. \tag{8}$$

The equilibrium yield minimizes the production cost, and the quantity of food produced is such that the price is equal to the marginal cost $\gamma(y,r)$. Let us denote as $y^R(r)$ and $F^R(r)$ the two equilibrium quantities. They satisfy

$$P(F^R) = \gamma(y^R,r) \text{ and } \frac{\partial \gamma}{\partial y}(y^R,r) = 0. \tag{9}$$

The situation $r = 0$ corresponds to a no-regulation situation with $\gamma(y,0) = c(y)/y$ so that $y^R(0) = y_0$ and $P(F^R(0)) = c(y_0)/y_0$. The quantity of food produced and the yield can be either increasing or decreasing with respect to the regulatory variable. Before considering some particular regulations, we first provide an analysis without further specifying the regulation. This degree of generality allows to show the underlying mechanisms at stake.

Any change of the regulation has the following effect on welfare, given by equation (4):

$$\begin{aligned} \frac{dW}{dr} &= \frac{\partial W}{\partial F}F^{R\prime} + \frac{\partial W}{\partial y}y^{R\prime} \\ &= \left[P(F) - \frac{c(y)}{y} - \beta\frac{b(0)-b(y)}{y}\right]F^{R\prime} \\ &+ \frac{F}{y^2}\left[c(y) - c'(y)y + \beta\left(b'(y)y + b(0) - b(y)\right)\right]y^{R\prime} \end{aligned} \tag{10}$$

At $r = 0$, the price is equal to the cost $c(y)/y$, and the yield is y_0 so that the derivative of welfare is

$$\frac{dW}{dr} = -\beta\frac{b(0)-b(y)}{y}F^{R\prime} + \left[\beta\left(b'(y)y + b(0) - b(y)\right)\right]\frac{F}{y^2}y^{R\prime}. \tag{11}$$

Whether a small increase of r will be beneficial or not depends on the sign of this expression. There is an unambiguous benefit from any reduction of the quantity of food produced (first term of 11). Whether an increase or a decrease of the yield is welfare enhancing depends on the density-yield curve.

If the monotonicity of the yield and the food production are aligned with the welfare-enhancing ones, a small increase in the regulation will be beneficial. If this is not the case, comparison of the two terms will be needed, and demand elasticity will play a crucial role.

A general expression of a threshold elasticity is

$$\tilde{\epsilon} = \left(1 + \frac{b'(y_0)y_0}{b(0) - b(y_0)}\right) \frac{\partial^2\gamma/\partial r\partial y}{y_0\,\partial^2\gamma/\partial y^2} \frac{\gamma}{\partial\gamma/\partial r} \tag{12}$$

The first factor represents the gain from an increase in the yield relative to the gain from a reduction of the food consumption. It is related to the shape of the density yield curve. It is null if the density yield curve is linear, positive if it is convex and negative if it is concave. The second factor is the rate of change of the yield with respect to the regulation, it is notably determined by the curvature of the cost function. And the last factor is the inverse of the rate of change of the cost $\gamma(y^R(r), r)$. In case of ambiguity about the merit of a small positive regulation, the demand elasticity should be compared with this ratio. The following table summarizes the possible cases.

Proposition 3 *The sign of the welfare effect of a small increase of the regulatory variable depends on the shape of the density-yield curve, as follows:*

		b concave ($y^* < y_0$)	*b convex* ($y^* > y_0$)
$y^{R\prime} < 0$	$F^{R\prime} < 0$	+	+ *if* $\epsilon \leq \tilde{\epsilon}$, - *otherwise*
	$F^{R\prime} > 0$	+ *if* $\epsilon \geq \tilde{\epsilon}$, - *otherwise*	-
$y^{R\prime} > 0$	$F^{R\prime} < 0$	+ *if* $\epsilon \leq \tilde{\epsilon}$, - *otherwise*	+
	$F^{R\prime} > 0$	-	+ *if* $\epsilon \geq \tilde{\epsilon}$, - *otherwise*

The Proof is in Appendix 6. A regulatory small change that modifies the yield in accordance to the optimal solution is welfare enhancing if the quantity produced decreases (the two shaded boxes) or if the elasticity of the demand is sufficiently close to zero ($\epsilon \geq \tilde{\epsilon}$). If production increases and the yield diverges from the optimal one, a small regulatory changes is indeed detrimental. In the two last cases, even though the yield moves in the wrong direction (compared to the optimal yield) the regulation can be welfare enhancing if the demand is sufficiently elastic. It show that a focus on the yield can be misguided when the demand elasticity is large in two types of situations: First, if the yield moves closer to the optimal one, the regulation is detrimental if production increases, as would be the case if a subsidy is implemented. Second, if the yield moves away from the optimal one, the regulation can still be welfare enhancing if production decreases sufficiently.

Another way to look at the trade-off would be to write the derivative of welfare at $r = 0$ grouping terms so that the effect of the regulation on farmland appears. Noting that $L' = (F'y - y'F)/y^2$, from equation (11), we obtain

$$\frac{dW}{dr} = \beta b'(y_0) L y^{R\prime} - \beta \left(\frac{b(0) - b(y)}{y} \right) L' \tag{13}$$

This expression emphasizes the trade-off between yield and farmed area, but it masks the role played by the demand price elasticity. However, this expression still allows us to obtain the following intuitive and reassuring result.

Proposition 4 *A sufficient condition for a small regulation to improve welfare is that both the yield and the farmed area decrease.*

At the laissez-faire equilibrium the difference between consumers surplus and production cost is maximized, and are not affected by a small change of the yield and farmed area by an envelop argument. For a small changes of r only the environmental effect matters, and it is positive if both the yield and farmed area decrease. It is true whatever the optimal solution might be.

Armed with these results, we can now consider several particular regulations.

4.2 Subsidizing wildlife-friendly farming

We begin by considering the consequences of a subsidy that would only hold on farmed land. Let us denote by s a subsidy on species specimen for farmland. This subsidy is constrained to be positive; that is, a tax on the specimen is not feasible. The profit of the representative land-owner is

$$\pi = PF - c(y)L + sb(y)L = PF - c(y)F/y + sb(y)F/y. \tag{14}$$

When choosing the yield for farmland, the land-owner does not consider the effect of land substitution between farmed and unfarmed land. He sets $y(s)$ and produces $F(s)$ so that

$$P(F) = [c(y) - sb(y)]/y \text{ and } c(y) - c'(y)y = s[b(y) - b'(y)y] \tag{15}$$

When the subsidy increases, the representative farmer reduces the yield and increases its production of food. The consequence is an increase of the total farmed area. Land-sparing is therefore not an option.

Corollary 1 *With a subsidy per specimen on farmland (and not on unfarmed area),*

- *If the density-yield curve is convex, the optimal subsidy is null.*
- *If the density-yield curve is concave, the optimal subsidy is null if $\epsilon < \tilde{\epsilon}$ and positive otherwise. The expression of the threshold is*

$$\tilde{\epsilon} = \left(1 + \frac{b'(y_0)y_0}{b(0) - b(y_0)}\right)\left(1 - \frac{b'(y_0)y_0}{b(y_0)}\right)\frac{c(y_0)}{y_0^2 c''}(< 0) \tag{16}$$

Proof. From the two equations (15), F^R is increasing and y^R is decreasing with respect to s. The situation corresponds to the third line of the table in proposition 3.

The expression of the threshold is obtained by inserting into the general expression (12) the relation $\partial\gamma/\partial s = -b(y)/y$ and the derivative of the yield at $s = 0$ (obtained from eq. (15)):

$$y^{R\prime}(0) = \frac{b - b'(y_0)y_0}{-c''y_0}$$

.

■

Subsidizing biodiversity of farmland has the adverse consequence of increasing the incentive to farm. If the species under consideration is very sensitive to the first increase of the yield (b convex), it is clearly detrimental to subsidize in-farm biodiversity. The gains in the farm cannot compensate the loss due to increased farmland. However, if the species is resistant to the implementation of farming (b concave), the gains from the reduction of the yield in farms are not fully compensated by farms' expansion if the demand for food is sufficiently inelastic. In that case, the food consumed does not increase much following the reduction of food price.

4.3 Implementation of natural reserves

Let us now consider the implementation of a natural reserve. This regulation would consist of setting $L = F/y$ the total farmed area. Equivalently, to better suit our general approach, the regulator can tax farmland. The regulatory variable is then the tax t. The profit of the representative land-owner is then

$$\pi = PF - (c(y) + t)L = \left[P - \frac{c(y) + t}{y}\right] F. \tag{17}$$

The quantity of food produced decreases and the yield increases with respect to the tax. The increase of the yield does not fully compensate the reduction of farmland.

Corollary 2 *When the regulator envisions taxing or subsidizing farmland,*

- *If the density-yield curve is convex, farming should be taxed.*
- *If the density-yield curve is concave, farming should be taxed if $\epsilon < \tilde{\epsilon}$ and subsidized otherwise. The expression of the threshold is*

$$\tilde{\epsilon} = \left(1 + \frac{b'(y_0)y_0}{b(0) - b(y_0)}\right) \frac{c(y_0)}{y^2 c''} \tag{18}$$

Proof.

With a tax on farmland, the marginal production cost of food is $\gamma(y, t) = (c(y) + t)/y$, the two quantities $y^R(t)$ and $F^R(t)$ satisfy

$$c'(y^R)y^R - c(y^R) = t \text{ and } P(F^R) = (c(y^R) + t)/y^R.$$

The quantity of food is decreasing and the yield is increasing with the regulatory variable. This case corresponds to the second line of the table in Proposition 3.

The particular expression of the threshold is obtained from equation (12) and the two following derivatives at $t = 0$:

$$\partial\gamma/\partial t = 1/y_0 \text{ and } y^{R\prime} = 1/c''$$

. ■

If the density-yield curve is convex, taxing farmland or implementing natural reserve is unambiguously good because it both reduces food consumption and increases the yield. Land is effectively spared, and the species gains more from this than it loses from the increased yield in farms.

If the density-yield curve is concave, in a first-best setting, it would be optimal to reduce the yield, which suggests that farmland should be subsidized. With an inelastic demand function, farming should indeed be subsidized. However, if the demand function is sufficiently elastic, farming should be taxed because the loss of biodiversity within farms is compensated by the overall reduction of food consumption. The expression of the threshold elasticity is a product of two factors: the first is the negative relative loss of biodiversity from the increased yield, and the second is the convexity of the cost function, which determines the sensitivity of the yield to an increase of the tax.

4.4 Taxing a dirty input

A last possibility considered would be to tax the input responsible for the loss of biodiversity. Here, we will not consider substitution among inputs; we only consider that the yield is determined by a quantity q of pollutant inputs. The function $q(y)$ is the quantity of inputs required to obtain a yield y It is null at zero, positive, increasing and convex. The function $b(y)$ is then an observed indirect relationship between the yield and the density that occurs via the quantity q.

If the regulator envisions taxing the input, the regulatory variable r is the tax, and the profit of farmers is

$$\pi = \left[P - \frac{c(y) + rq(y)}{y}\right] F$$

The yield decreases with the tax on the input, and the food produced is reduced. Farmed area unambiguously decreases with the input tax.

Corollary 3 *If the regulator can only tax or subsidize a pollutant input,*

- *If the density-yield curve is concave, the dirty input should be taxed.*
- *If the density-yield curve is convex, the dirty input should be taxed if $\epsilon < \tilde{\epsilon}$ and subsidized otherwise.*

The expression of the threshold elasticity is

$$\tilde{\epsilon} = \left(1 + \frac{b'(y_0)y_0}{b(0) - b(y_0)}\right)\left(1 - \frac{q'(y_0)y_0}{q(y_0)}\right)\frac{c(y_0)}{y_0^2 c''} \tag{19}$$

Proof. The two first-order conditions are

$$c(y^R) + rq(y^R) - (c' + rq')y^R = 0 \text{ and } P(F^R) = (c + rq)/y^R \tag{20}$$

The derivative of the yield with respect to r at $r = 0$ is $y^{R\prime} = \frac{1}{y_0}\frac{q - q'y_0}{c''}$, which is negative. The derivative of the cost is $\partial\gamma/\partial r = q(y_0)/y_0$. Therefore, the situation corresponds to the first (resp. third) line of the table in Proposition 3 for a tax (resp. a subsidy). Injecting these two derivatives into the general expression of the thresholds (12) gives the particular threshold (20).

■

If the density-yield curve is concave, both the reduction of the food consumed and the reduction of the yield go in the right direction, from a welfare perspective.

If the density-yield curve is convex, it would be optimal in a first-best setting to increase the yield and reduce the area farmed. However, if the demand for food is sufficiently elastic, it is optimal in a second-best setting to tax the dirty input. Such a tax induces a reduction of the food consumed that ensures that the area farmed does not increase too much and may decrease.

5 Discussions

Several important features that were not introduced in the model are likely to modify the results or their policy interpretation. Indeed, this theoretical analysis does not aim to provide definite answers about the optimal policy to protect a species; it mainly considers how the optimal farming technique and conservation policy depend upon the type of policy used. Three issues are briefly discussed: the role of input substitution, technical progress and irreversibility.

The density-yield curve observed is the result of a complex interaction between farming practices and the eco-system. Various farming practices can induce similar yields at different environmental costs. Indeed, organic farming can have a high yield, but it might require more work and knowledge than intensive farming. From a micro-economic perspective, this would mean that it is possible to substitue bidiversity degrading dirty inputs (e.g., pesticides and fertilizer) with less damaging ones (e.g., labor and knowledge). The meaningful economic evaluation of a technique is to determine the productivity of the various inputs.

The model should be extended by writing the yield and the density of the species as functions of a vector of input quantities. The optimal input combination would depend on the value of the species. The environmental effect of an input would be its direct effect on in-farm density plus its indirect effect via land use. The latter is related to the productivity of the input, so it would likely exhibit decreasing return to scale. Biodiversity preserving inputs have clear environmental benefits because they increase yield while preserving in-farm biodiversity. Whether biodiversity damaging inputs should be more intensively used would depend on whether their influence on the yield is sufficient to compensate for their in-farm environmental cost.

Substitution can be difficult to manage and can give rise to surprising consequences. For instance, if increasing the quantity of clean inputs increases the productivity of dirty ones, this can reinforce the case for their use. The analysis of policy would be affected by such substitution patterns in interesting ways because policies usually target some inputs and not others.

It is often argued that technical progress is a necessary ingredient to decouple economic growth from its environmental footprints, and, in particular, to increase food production while reducing the environmental externalities of farming. An interesting question related to the issue of input substitution is the direction of technical change and the orientation of agronomic research toward the productivity of certain inputs.

Finally, the ecological dynamic of the model should be developed. The long history of farming in Europe is partly responsible for the current environmental situation, and the currently observed density-yield curve is the result of past choices. It would be helpful to obtain dynamic trajectories of farming practices associated with the evolution of the species density. It would also help to understand the impact of the irreversibility of some habitat destruction on the trade-off between land-sparing and land-sharing. Whether the quasi-option value associated with this irreversibility (Henry; 1974; Arrow and Fisher; 1974) reinforces or reduces the case for land-sparing is an important research question.

6 Conclusion

This article has analyzed the trade-off between food production and nature conservation. The growth of the human population raises concern about the difficulty of ensuring food security and protecting the eco-system. It seems that highly productive techniques (e.g., intensive farming) can ensure the former but sacrifice the latter. This is not necessarily so if these techniques allow land to be spared for nature.

It has been shown that the optimal yield can be increasing with respect to the value of the threatened species. If this species is highly sensitive to the first increase of the yield, it is optimal to protect it to increase yield and spare land. However, wildlife-friendly farming, even if it is low yield, is not necessarily associated with expanded farmland if the demand for food is sufficiently elastic. So that a combination of agri-environmental schemes and natural reserves can be the optimal solution.

Second-best policies that cannot directly act upon both the yield and the farmed area will be welfare enhancing in certain conditions on the density-yield curve and the demand elasticity. For instance, if the density of the species is decreasing with respect to a dirty input, it is optimal to tax this input and reduce the yield even in cases in which it would be optimal in a first-best setting to increase yield. This is because the decrease of the yield is compensated by a decrease of the food consumed, which ensures that farmed area does not increase significantly. However, if the demand function is inelastic, then it may be optimal to subsidize a dirty input to spare land.

The analysis of the second-best setting, although highly stylized, shows that policy recommendations that are a priori true in a first-best setting are not necessarily true in second-best settings. People inspired by conservation purposes should not jump to the conclusion that a certain type of agriculture should be promoted because this type of agriculture is part of a first-best strategy. If the regulation is incomplete, it may be welfare enhancing to promote a priori bad agricultural practices.

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Appendix

Proof of Proposition 1

Proof.

1. The optimal quantity of food satisfies

$$P(F^*) = \frac{c(y^*)}{y^*} + \beta \frac{b(0) - b(y^*)}{y^*}$$

The derivative of the right-hand side with respect to β is, by the envelop theorem, $[b(0) - b(y^*)]/y^*$, which is positive. Because the price function is a decreasing function, the optimal quantity of food is decreasing w.r.t. β.

2. The derivative of the right-hand side of equation (6) is $-b''(y)$, and the right-hand side is null at $y = 0$.

- If $b(.)$ is concave,

  the marginal environmental damage is increasing ($-b'' > 0$). Because it is null at $y = 0$, it is positive. At the optimum, a marginal change of β would increase the marginal environmental damage and subsequently decrease the optimal yield. (at the interior optimum, the second-order condition is satisfied, and the effect of a change of β on the optimal yield is the opposite of its effect on the right-hand side of 6).

- If $b(.)$ is convex,

  the right-hand side of 6 is decreasing and null at zero; therefore, it is negative. The optimal yield is increasing with respect to β.

■

Proof of Proposition 2

Proof. The optimal farmed area is $L^*(\beta) = F^*(\beta)/y^*(\beta)$.

If the density-yield curve is convex, L^* is decreasing w.r.t. β because F^* is decreasing and y^* is increasing w.r.t. β.

If the density-yield curve is concave, let us write the β elasticity of the farmed area

$$\frac{\beta L^{*\prime}}{L^*} = \frac{\beta F^{*\prime}}{F^*} - \frac{\beta y^{*\prime}}{y^*} \tag{21}$$

. Using equation (5) and (6) gives

$$\frac{\beta F^{*\prime}}{F^*} = \frac{\beta(b(0) - b(y))}{c(y) + \beta(b(0) - b(y))}\frac{1}{\epsilon} \text{ and } \frac{\beta y^{*\prime}}{y^*} = \frac{\beta}{y^*}\frac{b'y^* + (b(0) - b(y^*))}{(c'' - \beta b'')y^{*2}}$$

.

Inserting these two equations into the expression (21) gives the result. ■

Proof of Proposition 3

Proof.

The price of food is equal to the marginal cost. Taking the derivative of the first equation in (9) gives (by the envelop theorem) $P'F^{R\prime} = \partial\gamma/\partial r$, so

$$F^{R\prime} = \epsilon F \frac{\partial\gamma/\partial r}{\gamma(y, r)}.$$

Then, inserting the above equation into equation (13), the derivative of welfare is

$$\begin{aligned} \frac{dW}{dr} &= \beta\frac{F}{y}(b(0) - b(y))\left[-\epsilon\frac{\partial\gamma/\partial r}{\gamma(y,r)} + \left(1 + \frac{b'(y)y}{b(0) - b(y)}\right)\frac{y^{R\prime}}{y}\right] \\ &= \beta\frac{F}{y}(b(0) - b(y))\frac{\partial\gamma/\partial r}{\gamma(y,r)}(\tilde{\epsilon} - \epsilon) \qquad \text{using (12).} \end{aligned} \tag{22}$$

Let us consider that $\partial\gamma/\partial r$ is positive; therefore, $F^{R\prime}$ is negative.

- If $b(.)$ is concave, the effect of the yield $b(0) - b(y) + b'(y)y$ is negative.

  If $y^{R\prime}$ is negative, then the two terms in the expression (13) of the derivative of welfare are positive, and a small increase of r has a positive effect.

  If $y^{R\prime}$ is positive, the threshold $\tilde{\epsilon}$ is negative, and from (22), the derivative of welfare is positive if $\epsilon < \tilde{\epsilon}$ and negative otherwise.

- If $b(.)$ is convex, the effect of the yield $b(0) - b(y) + b'(y)y$ is positive. A similar reasoning gives the results for the second row of the table.

If $\partial\gamma/\partial r$ is negative, $F^{R\prime}$ is positive. Symmetrical reasoning could be applied to obtain the last two lines of the table.

■